

which the syndics of the Cambridge University Press have intrusted to me to complete and edit. In reading the great number of memoirs relating to the subject I have been much struck by the want of a clear and accurate terminology in both theoretical and practical elasticity. I have been forced to the conclusion that the great discrepancy, which is often to be found between theoretical and practical results, is in some measure due to the want of this terminology (*e.g.* the extreme looseness of the term "limit of elasticity"). I find it needful for the purposes of the above work to adopt such a terminology, but before doing so it would be extremely valuable to have the opinion of some of our leading elasticians on the terms I venture to propose. I should be very glad of any suggestions, through the columns of NATURE, towards a definite and uniform terminology.

I am particularly dissatisfied with the term "limit of superimposition." It is exceedingly clumsy. Other possible terms are—"limit of superposable stress," "linear limit," and "limit of constant slope," the last two phrases having reference to the fact that the stress-strain curve at this limit ceases to be a straight line. With regard to this limit of superimposition I may remark that it may arise from one of two causes—(1) The strain components become so large that we cannot neglect the squares of small quantities, or the stress components can no longer be taken proportional to those of strain. This might happen before permanent set. (2) Permanent set may arise which does not follow the generalised Hooke's Law. This seems the more probable case, and has been adopted below. Prof. Kennedy tells me that he thinks when a body has been reduced to a state of ease that the superior elastic limit and the limit of superimposition coincide.

It has been proposed, I believe, to term that limit of stress at which bars begin locally to "thin down" the limit of viscosity. The "limit of uniform strain" is not altogether satisfactory or quite suggestive of this peculiar viscosity. "State of maximum stress" might perhaps serve the purpose, were one quite sure that this state always coincides with the viscous limit.

In the following remarks I have been much assisted by Prof. J. Thomson's epoch-making paper in the *Cambridge and Dublin Mathematical Journal* for 1848, and even more by Prof. Alex. B. W. Kennedy's paper on Riveted Joints in the *Proceedings of the Institution of Mechanical Engineers*, April, 1881 (especially pp. 208-213).

We have first to distinguish between two classes of materials. In the one we may suppose the particles to be in a state of internal stress before any external force is applied; in this case any, the least, external stress will probably produce permanent set. If this stress be removed and then reapplied, after one or two trials it will cease to produce permanent set, or at least the permanent set will be extremely small as compared with the elastic strain. We thus need a term to mark that state of the body when external stress does not produce permanent set owing to the existence of internal stress. This might perhaps be termed the *state of ease*. Many discordant results with regard to the constants of elasticity are not improbably due to the fact that the ratio of stress to strain has been measured before the material has been reduced to a *state of ease*. In the second class of materials we may suppose this state of ease to exist before the application of any stress. Supposing a body to be in its state of ease, there will then exist two limits, one on one side, and one on the other of the unstrained shape, which may be termed the *inferior and the superior limits of perfect elasticity*. Any external stress which does not produce a strain exceeding these limits will not give rise to permanent set. These inferior and superior limits of perfect elasticity mark, as a rule, the range covered by the usual mathematical theory. Within these limits it is generally safe to assume that the components of internal stress are proportional to the components of strain. In some cases, *i.e.* cast iron, where, however, it is difficult to produce the *state of ease*, this does not seem to be accurate—the stress and strain components appear never to be proportional.

In most materials the range of perfect elasticity is not large. An external stress, which is by no means nearly equal to that which is required to produce rupture will give rise to a permanent set. Thus permanent set in some materials will commence at a stress only  $\frac{3}{4}$  to  $\frac{1}{2}$  of the stress that those materials are capable of standing. Thus beyond the limit of elasticity we have first a range of stresses, which produce strains partly elastic and partly permanent. The strain in this range might still remain proportional to the stress; the permanent is yet small as com-

pared with the elastic part of the strain. This range is bounded, however, by a stress for which the strain ceases to be proportional to the stress. In other words, the "generalised" Hooke's law is no longer applicable. Up to this point, if we are merely desirous of finding the strain produced by any system of statical stress, the mathematical equations of elasticity will apply, supposing, as seems probable, that the elastic constants do not alter, owing to the permanent set. Those equations would not of course be valid if we wished to find the strain in the body, if the stress were altered, nor would they suffice to treat vibratory motions capable of producing permanent set. This limit, which is that at which the *ut tensio sic vis* principle ceases, requires a name. It might perhaps be termed the *limit of superimposition*. That is to say, if a certain addition to this limiting stress produced a certain increase of strain, and a second addition another increase, these increments of stress, if superimposed would not produce the sum of the strain increments. It might at first sight appear more direct to term it the modular limit, or the limit of Hooke's law, but it would seem that, after this limit is passed, Hooke's law, probably with the same modulus, applies to so much of the strain as is elastic strain; in fact at the limit of superimposition it is the permanent set part of the strain which ceases to obey Hooke's law. In some materials the limits of perfect elasticity and of superimposition may coincide. At the latter limit the permanent set is still in some cases only one-twenty-fifth of the total strain. Neither of the limits above considered is *commercially* treated as the limit of elasticity. This is the point at which the material "breaks down," that is to say, the stress being continually increased, a strain is obtained which would be preserved by replacing the stress by one very much less. The material is unable to balance the stress upon it. If the stress be maintained the strain will suddenly increase by a considerable amount (without the stress being increased). This remarkable limit, it has been suggested by Mr. Tweddell, should be termed the *limit of fatigue*. The limit of fatigue being past, a small proportion of the strain, namely, so much as corresponds to the modulus, is elastic, the greater part is permanent set.

In the case of bars of iron subjected to longitudinal pull, if the stress be increased beyond the limit of fatigue, another limiting strain is reached, namely, one at which local contraction begins, or the bar commences to draw out at some point, *i.e.* the strain ceases to be uniform. The material now begins to act as if it were "viscous," and it would be convenient to describe this state as that of *viscosity*, had not this name been appropriated to that permanent set which may be produced by the application for a long period of a stress well within the limits of perfect elasticity. Closely associated, if not the same, with this *limit of uniform strain* is the state of maximum load. From this point onwards, as the strain increases the load decreases, till the breaking load is reached with a magnitude below that of the maximum load. To distinguish one from the other requires a special manipulation. As a rule, what is meant by the *absolute or breaking strength* is probably the *maximum load*, for if this load was allowed to remain, the bar would break under it. It might perhaps be convenient, however, to speak of one as the *maximum* and the other as the *terminal load*. With the terminal load the "*elastic life*" of the material is concluded. It must be remembered that owing to the bar locally thinning down, the stress per unit area at the terminal load is greater than the stress per unit area at the maximum load.

Such are the limits for which it is useful that a terminology should be established. I shall be extremely glad if any of the readers of NATURE, who happen to be elasticians, will suggest more concise phraseology.

KARL PEARSON

University College, February 14

#### Civilisation and Eyesight

I HAVE been interested in Lord Rayleigh's note on "Vision," and would offer my mite on the subject.

I have no doubt that brilliancy of image and power of distinguishing largely depend on definition. The brilliancy does so for the same reason as that which induces an artist to heighten colour-effects by sharp contrasts. In the same way, if we seek to decide if two colours are alike, we place them in immediate contact with a sharp edge. Details are best seen with a telescope when the images are sharp and untroubled. When slight tremors are in the air, and the image is rapidly displaced in all directions, so that what we see is the resultant

of many rapidly succeeding impressions, then tints are graded into one another at the edges, and we lose the power of distinguishing detail.

I can give, fortunately, a case in point. My eyes are affected with a small amount of astigmatism. It does not affect general vision for ordinary purposes, nor, of course, the definition of single lines; but, when I use appropriate lenses, the whole scene becomes brighter and more cheerful, and I see details. The bark of a tree is a perfectly different object with and without them. With them it is like a good photograph; without them, like many pictures. Formerly, in addition to the cylindrical surface, I required a slight spherical concave, and I was disposed to place the increased general brilliancy of the view mainly to the reduction of size, but I now use plano-cylindrical lenses for distant vision, and it is evident that the brilliancy is solely due to the better definition.

I would, lastly, suggest for Lord Rayleigh's consideration the question whether the change of focus of his eyes in faint light is not partly, at all events, due to change in the colour of the light. I know that there is such a change with me, but I have long had reason to believe that colour affects my vision.

J. F. TENNANT

37, Hamilton Road, Ealing, W., February 7

THOSE who have compared Lord Rayleigh's letter in NATURE of February 12 with that of Mr. Brudenell Carter on February 26 will have observed an inconsistency occasioned by a slip of the pen.

The latter says: "The commonly accepted standard of normal vision is satisfied by deciphering letters the parts of which subtend visual angles of one minute. . . ." Also, Prof. McKendrick states that "The smallest visual angle in which two distinct points may be observed is 60 seconds."

According to Lord Rayleigh, however, "A double star cannot be fairly resolved unless its components subtend an angle exceeding that subtended by the wave-length of light at a distance equal to the aperture. If we take the aperture of the eye as  $1/5$ th inch, and the wave-length of light as  $1/40,000$ th inch, this angle is found to be about two minutes." In the case of a small angle the aperture divided by the distance is approximately equal to the arc divided by the radius or to the circular measure of the angle. Hence in the present case we have  $\frac{1/40,000}{1/5}$ th inch =

$\frac{1}{8000}$  radian or  $\frac{206,265}{8000} = 25.8$  seconds nearly, instead of the two minutes accidentally stated by Lord Rayleigh.

This minimum value seems to show some mistake in Ehrenberg's experiments on vision, and is about half of that found by Helmholtz for the best of twelve observers.

March 10

SYDNEY LUPTON

[Mr. Lupton is quite right. By a stupid blunder I said about two minutes, when I should have said about half a minute.—RAYLEIGH.]

THERE is a defect of eyesight common among the natives of India known as "rātandhi," i.e., "night blindness." Persons affected with this have either ordinary powers of vision by daylight, or else powers so little less than ordinary as to feel no inconvenience, so that usually no defect is noticeable; whilst in feeble twilight their sight fails in the most extraordinary way, and in the dusk they become (in bad cases) practically blind. Of course there are all degrees of this affection; but the strongly-marked cases alone are likely to attract attention.

By medical men in India this affection is said to occur most among men living on a low diet (chiefly of cereals), and the usual palliative treatment is to prescribe a meat diet.

This affection is rarely noticeable among Europeans in India, though I have sometimes noticed marked differences of clearness of sight among them also amounting to slight "night-blindness." Lord Rayleigh's case of short-sightedness in twilight and in the dusk seems to be a mild case of this sort (see NATURE, February 12, p. 340).

ALLAN CUNNINGHAM

#### The Pupil of the Eyes during Emotion

ALTHOUGH further observations are required, there seems to be a more or less general assent as to the influence of the emotions on the pupils of the eyes. Mr. Clark, in his letter to your journal (vol. xxxi. p. 433), has rightly quoted Gratiolet, who

says that in sudden astonishment or fear the whole system becomes paralysed, and at the same time the pupils dilated. In anger, on the other hand, when the whole body is roused into action, the pupils become contracted: "Les pupilles sont énormément dilatées dans l'épouvanté, tandis qu'elles sont toujours contractées dans le colère." This was, however, said many years before by the celebrated Harvey, who, in his discourse on the circulation of the blood, written in 1628, says: "In anger the eyes are fiery, and the pupils contracted" ("Irâ rubent oculi, constringit pupilla").

I should myself think that a narrow pupil evinces a more active mental state, as it is this condition which is present when the eye is accommodated to regard with attention a near object, whilst, on the other hand, when gazing out into distance, the pupils are wider, and the mental mood is more passive and contemplative.

In my parrot the size of the pupil is a very excellent measure of its frame of mind. When angry the pupil becomes minutely contracted, whereas when the bird is sympathetic and amiable the pupils become as widely dilated. Balzac, with other novelists, have depicted the state of the pupils when describing the various emotions and passions. The former in pourtraying a saintly woman kneeling before the altar, says: "The pupil of the eye, endued with great contractility, appeared then to expand and draw back the blue of the iris until it formed no more than a narrow circle. What force was that arising in the depths of the soul which so enlarged the pupils in full daylight and obscured the azure of those celestial eyes?" Darwin speaks doubtfully, but rightly demands more observations on the subject.

SAMUEL WILKS

Grosvenor Street, March

#### Auroræ

AFTER a long and remarkable absence of aurora, which, from a letter in your columns of February 19 (p. 360) does not appear to have been confined to these more southerly latitudes, we were favoured last evening with a beautiful, though somewhat transient display. It was about 9<sup>25</sup> p.m. when I first noticed a long band or belt of light above the northern horizon. At first it was ill-defined, with little change of position, but in about twenty minutes it became more luminous and the characteristic streamers suddenly made their appearance, shooting upwards, sometimes from above, sometimes from below the belt of light, which for a few seconds changed into a double arch. Some of these streamers rose as distinct columns, showing the usual prismatic hues, one in particular being noticeable as traversing the inverted W of Cassiopeia, another forming a fan-like terminus to the luminous region, but all confined to a low altitude, bounded on the north-west by Perseus, and on the north-east by Vega, then rising. It may be well to observe that on the same day (the 15th) a large sun-spot had just reached the central meridian, and was beginning to show signs of great disturbance.

E. BROWN

Further Barton, Cirencester, March 16

#### Injuries caused by Lightning in Venezuela

IN answer to Mr. von Danckelman's inquiry as to the use of lightning-rods and the frequency of accidents from lightning in the tropics (NATURE, December 11, 1884, p. 127), I beg leave to offer the following information referring to Venezuela, where I have been residing ever since 1862:

Thunderstorms are very frequent during the rainy season. They break out generally in the afternoon, about the time of the daily maximum of heat, whilst they are extremely rare in the morning (I only witnessed one case) and during the night. Statistics of accidents do not exist, nor are there many lightning-rods in use (in Caracas about half a dozen). But there are certain regions where the former are far from being uncommon, as, for instance, the country around the Lake of Valencia and the plains or *llanos* to the north of the Orinoco. In these a considerable number of cattle are killed by lightning every year, and I know also of several cases where houses were destroyed and people killed. The herds of cattle crowd together as soon as a thunderstorm begins, and the animals remain during the whole time with their heads down to the ground, thus avoiding instinctively that their pointed horns should act as lightning-conductors.

In the neighbourhood of Maracay, at the eastern end of the Lake of Valencia, accidents occur almost every year. A very